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Ex 1.1.

Q. 1. Show that the relation R in the set $\{1,2,3\}$ given by $R = \{(1,2),(2,1)\}$ is symmetric but neither reflexive nor transitive.

Soln. It is given that $A = \{1,2,3\}$

A relation R on A is defined as $R = \{(1,2),(2,1)\}$

Here, we see that $(1,1),(2,2),(3,3) \notin R$

$\therefore$ R is not reflexive.

Now, $(1,2) \in R$ and $(2,1) \in R$

$\therefore$ R is symmetric

Now, $(1,2)$ and $(2,1) \in R$ but $(1,1) \notin R$

So, R is not transitive.

Hence, R is symmetric but neither reflexive nor ~~transitive~~ transitive.

Q. 2. Show that the relation R in the set A of all the books in a library of ~~books~~ college, given by $R = \{(x,y): x$ and y have same number of pages$\}$, is an equivalence relation.

Soln. Here, we see that set A is the set of all books in the library of a college and $R = \{(x,y): x$ and y have the same number of pages$\}$.

Now, R is reflexive, since $(x,x) \in R$ [$\because$ x and x has the same number of pages]

Let $(x,y) \in R$
$\Rightarrow$ x and y have the same number of pages
$\Rightarrow$ y and x have the same number of pages.
$\Rightarrow (y,x) \in R$. So, R is symmetric.

Now, Let $(x,y) \in R$ and $(y,z) \in R \Rightarrow (x,z) \in R$
$\Rightarrow$ x and y have same number of pages and y and z have same number of pages $\Rightarrow$ x and z have same number of pages.
$\therefore$ R is transitive. Hence, R is an equivalence relation.

Q.3. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ is given by $R = \{(a, b) : |a-b| \text{ is even}\}$, is an equivalence relation. Show that all the elements of $\{1, 3, 5\}$ are related to each other and all the elements of $\{2, 4\}$ are related to each other, but no element of $\{1, 3, 5\}$ is related to any element of $\{2, 4\}$.

Soln: It is given that $A = \{1, 2, 3, 4, 5\}$, $R = \{(a, b) : |a-b| \text{ is even}\}$.

Let $a \in A \Rightarrow |a-a| = 0$, which is even, $\forall a$.

So, R is reflexive.

Let $(a, b) \in R \Rightarrow |a-b|$ is even $\Rightarrow |-(b-a)| = |b-a|$ is also even

$\Rightarrow (b, a) \in R$. So, R is symmetric.

Now, let $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow |a-b|$ is even and $|b-c|$ is even.

$\Rightarrow (a-b)$ is even and $(b-c)$ is even.

$\Rightarrow (a-c) = (a-b) + (b-c)$ is even.

[$\because$ Sum of two even integers is even]

$\Rightarrow |a-c|$ is even $\Rightarrow (a, c)$ is even.

So, R is transitive. Hence, R is an equivalence relation.

Now, all the elements of the set $\{1, 3, 5\}$ are related to each other as all the elements of this set are ~~even~~ odd. So, the modulus of the difference between any two elements will be even.

Similarly, all the elements of the set $\{2, 4\}$ are related to each other as all the elements of this set are even. So, the modulus of the difference between any two elements will be even.

Also, no element of the set $\{1, 3, 5\}$ can be related to any element of $\{2, 4\}$ as all elements of $\{1, 3, 5\}$ are odd and all elements of $\{2, 4\}$ are even. So, the modulus of the modulus of the difference between the two elements (from each of these two subsets) will not be even.

Proved.